Problem Solving 5

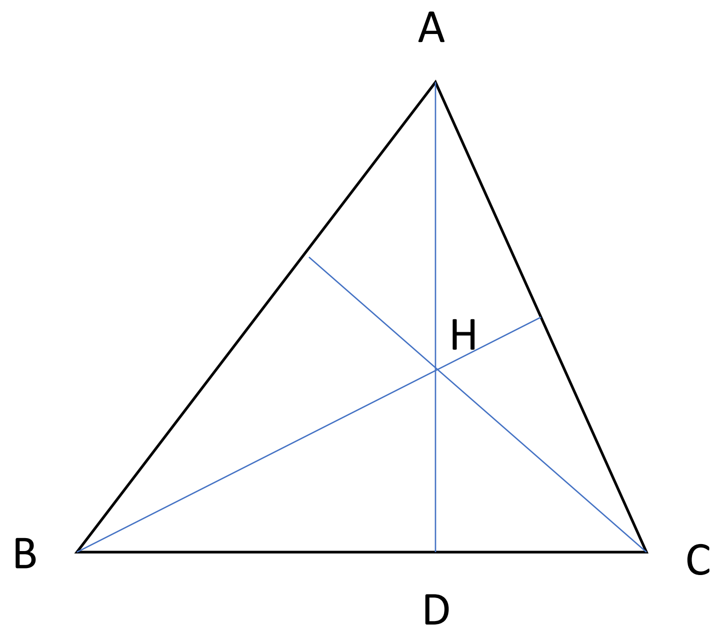
Lecture 16 Apr 25, 2021

- Q1 (leftover from last week). Find the solutions (x, y) of the equation

$$2x^2y^2 + y^2 = 26\,x^2 + 1201$$

in natural numbers.

- Q2. In the triangle ABC, AD is the height of A over BC and H is the intersection point of the three heights. If AD=4, BD=3, and CD=2, find the length of HD



- Q3. What is the highest power of 2 that divides $3^{512} - 1$?

- Q4. Natural numbers 1, 2, 3,…, 2021 are written on a board. In each turn, we replace two of them, say $x, y,$ with $|x-y|$, and we repeat this until one number is left. What can you say about the last number:

- It is always a multiple of 4
- It is always odd
- It is equal to 1 modulo 4
- It is always even
- None

- Q5. In how many ways we can put 10 coins in a row so that no adjacent coins are “head” up.

(Replace 10 with arbitrary n and solve it again)

- Q6. In a triangle ABC, the line that connects A to the mid-point of BC is orthogonal to angle bisector of A. If the side lengths of ABC are three consecutive natural numbers, what can be said about its perimeter.

- Q7. Find the number of rational numbers x between 0 and 2021 such that

$$3x^3 + 10x^2 - 3x$$

is an integer.

- Q8. Suppose $a_1, a_2, \dots, a_5$ are a permutation of 1,2,…,5. What is the max possible value of

$$a_1 a_2 + a_2 a_3 + \cdots + a_5 a_1$$

- (Try to solve the problem for arbitrary n instead of 5).